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Quantum computation with FQHE quasiparticles

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Abstract. We propose an approach to implementing anyonic quantum computation in systems of antidots in the two-dimensional electron liquid in the FQHE regime. The approach is based on the adiabatic transfer of FQHE quasiparticles in the antidot systems, and uses their fractional statistics to perform quantum logic. Advantages of our scheme over other semiconductor-based proposals of quantum computation are the energy gap in the FQHE liquid that suppresses decoherence, and the topological nature of quasiparticle statistics that makes it possible to entangle two quasiparticles without their direct dynamic interaction.

Introduction

"Topological" quantum computation with anyons has been suggested as a way of implementing intrinsically fault-tolerant quantum computation [1–4]. Intertwining of anyons, quasiparticles of two-dimensional electron system (2DES) with non-trivial exchange statistics, induces unitary transformations of the system wavefunction that depend only on the topological order of the underlying 2DES. These transformations can be used to perform quantum logic, the topological nature of which is expected to make it more robust against environmental decoherence.

The aim of this work is to propose specific and experimentally feasible approach for implementation of basic elements of the anyonic quantum computation. The approach is based on adiabatic transport of the Fractional Quantum Hall Effect (FQHE) quasiparticles in systems of quantum antidots. An antidot is a small hole in the 2DES produced by electron depletion, which localizes FQHE quasiparticles at its boundary due to combined action of the magnetic field and the electric field created in the depleted region. If the antidot is sufficiently small, the energy spectrum of the antidot-bound quasiparticle states is discrete, with finite excitation energy Δ. When Δ is larger than temperature T, modulation of external gate voltage can be used to attract quasiparticles one by one to the antidot [5, 6]. In this regime, adiabatic transport of individual quasiparticles in the multi-antidot systems can be used to perform quantum logic, in close analogy to adiabatic transport of individual Cooper pairs in systems of small superconducting islands in the Coulomb blockade regime [7]. In what follows, we describe a specific design of such logic gates and discuss their parameters.

1. FQHE qubit and logic gates

In analogy to Cooper-pair qubits [7–9], information in the FQHE qubits can be encoded by the position of a quasiparticle in the system of two antidots. In this case, the FQHE qubit (Fig. 1) is the double-antidot system gate-voltage tuned near the resonance, where the energy difference ε between the quasiparticle states localized at the two antidots is small, $\varepsilon \ll \Delta$. At energies smaller than Δ, dynamics of such double-antidot system is equivalent to the dynamics of a common two-state system (qubit). The states localized at

the two antidots are the $|0\rangle$ and $|1\rangle$ states of the computational basis of this qubit. The gate electrodes of the structure can be designed to control separately the energy difference ε and the tunnel coupling Ω of the resonant quasiparticle states. In the existing experiments [5, 6] demonstrating transport of individual quasiparticles, the gate voltages were applied through a combination of a backgate and front gates etched into the 2DES. Both types of gates are not optimal for the antidot operation as qubit. The global backgate can not address selectively individual antidots, while the etched-in front gates create edges which support low-energy excitations and thus introduce decoherence in the antidot dynamics. An antidot structure optimized for qubit operation should use front gates that do not deplete 2DES, and therefore do not create edges. With such gates, application of the time-dependent gate voltages to individual antidots, and to the 2DES region between the antidots, should make it possible to vary the qubit parameters ε and Ω in a wide range, and as a result, to perform arbitrary single-qubit transformations.

Fig. 1. Schematic structure (a) and energy profile (b) of the double-antidot FQHE qubit. Solid (dashed) lines (in (b), horizontal lines) indicate the edges of the incompressible electron liquid when the quasiparticle is localized at the right (left) antidot. Displacement of the electron liquid is quantized due to quantization of the single-particle states circling the antidots.

The most natural approach to construction of two-qubit gates with the FQHE qubits is to use fractional statistics [10, 11] of the FQHE quasiparticles. Due to this statistics, intertwining of the two quasiparticle trajectories in the course of time-evolution of the two qubits realizes controlled-phase transformation. Precise result of this operation depends on the nature of the FQHE state. In this work, we discuss the most basic and robust Laughlin state with the filling factor $\nu = 1/3$, when the quasiparticles have abelian statistics and intertwining of trajectories leads to multiplication of the state wavefunction by the phase factor $e^{\pm 2\pi i/3}$. The sign of the phase depends on the direction of magnetic field and direction of rotation of one quasiparticle trajectory around another.

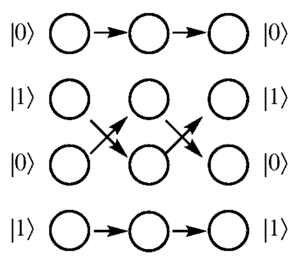

Fig. 2. Antidot implementation of the two-qubit controlled-phase gate. The states $|0\rangle$ and $|1\rangle$ are the computational basis states of the two qubits. The arrows show the quasiparticle transfer processes during the gate operation.

Possible structure of the controlled-phase gate is shown in Fig. 2. The columns of four antidots contains two qubits, and arrows denote trajectory of quasiparticle transfer through the system. The transfer leads to transformation of the quantum state of two qubits

and its shift from the gate input (left column in Fig. 2) to the output (right column). The quasiparticle transfer can be achieved by the standard adiabatic level-crossing dynamics. If a pair of antidots is coupled by the tunnel amplitude Ω, gate-voltage induced variation of the energy difference ε through the value $\varepsilon = 0$ (slow on the time scale Ω^{-1}) leads to the transfer of a quasiparticle between these antidots. Correct operation of the controlled-phase gate in Fig. 2 requires that the gate voltage pulses applied to the antidots are timed so that the state of the upper qubit is propagated at first halfway through the gate, then the state of the lower qubit is propagated through the whole gate, and finally the state of the upper qubit is transferred to the output. In this case, if the quasiparticle of the upper qubit is in the state $|1\rangle$, trajectories of the quasiparticle propagation in the lower qubit encircle this quasiparticle, and the two states of the lower qubit acquire additional phase difference $\pm 2\pi/3$, conditioned on the state of the upper qubit. We take the direction of magnetic field to be such that the state $|1\rangle$ of the lower qubit acquires a positive extra phase $2\pi/3$. Assuming that parameters of the driving pulses are adjusted in such a way that dynamic phases accumulated by the qubit states are the multiple integers of 2π, the evolution matrix P of the gate can be written as

$$P = \mathrm{diag}\,[1, 1, 1, e^{2\pi i/3}] \tag{1}$$

in the basis of the four gate states $|00\rangle$, $|01\rangle$, $|10\rangle$, $|11\rangle$. Controlled-phase gate (1) combined with the possibility of performing an arbitrary singe-qubit transformation is sufficient for universal quantum computation [12]. For instance, a combination of the two gates (1) with single-qubit gates can reproduce the usual controlled-NOT gate.

2. Estimates and discussion

The basic set of conditions necessary for correct operation of the FQHE qubits and gates described above can be summarized as $T \ll \varepsilon$, $\Omega \ll \Delta$. The antidot excitation energy Δ is estimated as $\Delta \simeq \hbar v/r$, where r is the antidot radius and $v \simeq 10^4 \div 10^5$ m/s [13] is the velocity of quasiparticle motion around the antidot. This means that at temperatures $T \simeq 0.1$ K the radius r should be smaller than 100 nm. Since the tunnel coupling Ω decreases rapidly with the distance d between the antidots, $\Omega \propto \exp\{-eBd^2/12\hbar\}$ [14], the fact that it should remain at least larger than T means that the distance between the tunnel-coupled antidots should not exceed few magnetic lengths $l = (eB/\hbar)^{1/2}$, where $l \simeq 10$ nm for typical values of the magnetic field B. Although these requirements on the radius r and antidot spacing d can in principle be satisfied with the present-day fabrication technology, they present a formidable challenge. It should be noted that these requirements are not specific to our FQHE scheme, but characterize all semiconductor solid-state qubits based directly on the quantum dynamics of individual quasiparticles, and not collective degrees of freedom (as in the case of superconductors). On the other hand, there are two advantages of the FQHE approach. First is the energy gap of the FQHE liquid that suppresses quasiparticle excitations and associated qubit decoherence in the bulk of the 2DES. The decoherence mechanisms for the FQHE qubits are localized at the 2DES edges and in the gate electrodes, and to a large extent can be controlled through the system layout. The second advantage is the topological nature of statistical phase that allows to entangle qubits without their direct dynamic interaction. This should lead to a much simpler design of the FQHE quantum logic circuit in comparison, e.g., to the superconducting qubits, where control of the qubit-qubit interaction presents a difficult problem.

Acknowledgements

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